Diploma Programme subject outline–Group 5: mathematics			
School name	**Gymnazium a SOS Rokycany**	**School code**	**061768**
Name of the DP subject *(indicate language)*	**Mathematics: applications and interpretation SL and HL (English)**		
Level *(indicate with X)*	Higher [X] Standard completed in two years [X] Standard completed in one year * [ ]		
Name of the teacher who completed this outline	**Tomas Trhlik**	**Date of IB training**	**2022**
Date when outline was completed	**Updated 08/2024**	**Name of workshop** *(indicate name of subject and workshop category)*	**Maths AI, Cat 1**

* All Diploma Programme courses are designed as two-year learning experiences. However, up to two standard level subjects, excluding languages ab initio and pilot subjects, can be completed in one year, according to conditions established in the *Handbook of procedures for the Diploma Programme*.

1. Course outline

– Use the following table to organize the topics to be taught in the course. If you need to include topics that cover other requirements you have to teach (for example, national syllabus), make sure that you do so in an integrated way, but also differentiate them using italics. Add as many rows as you need.

– This document should not be a day-by-day accounting of each unit. It is an outline showing how you will distribute the topics and the time to ensure that students are prepared to comply with the requirements of the subject.

– This outline should show how you will develop the teaching of the subject. It should reflect the individual nature of the course in your classroom and should not just be a “copy and paste” from the subject guide.

– If you will teach both higher and standard level, make sure that this is clearly identified in your outline.

	Topic/unit	Contents	Allocated time One class is 45 min. In one week there are 3 SL/HL classes + 1 HL class.
Year 1	Topic 1: Number and algebra	**SL 1.5** Laws of exponents with integer exponents. **SL 1.1** Operations with numbers in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer **SL 1.5** Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.	CHAPTER 1 SL
	Topic 1: Number and algebra Topic 2: Functions	**AHL 1.10** Simplifying expressions, both numerically and algebraically, involving rational exponents. **AHL 1.9** Laws of logarithms. **AHL 1.11** The sum of infinite geometric sequences. **AHL 2.10** Scaling very large or small numbers using logarithms. Linearizing data using logarithms to determine if the data has an exponential or a power relationship using best-fit straight lines to determine parameters. Interpretation of log-log and semi-log graphs.	CHAPTER 1 HL
	Topic 1: Number and algebra	**SL 1.2** Arithmetic sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. Applications. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life. **SL 1.3** Geometric sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for the sums of geometric sequences. Applications. **SL 1.4** Financial applications of geometric sequences and series.	CHAPTER 2 SL
	Topic 3: Geometry and trigonometry	**AHL 3.10** Concept of a vector and a scalar. Representation of vectors using directed line segments. Unit vectors, base vectors. Components of a vector, column representation The zero vector and opposite vector. Position vectors.	CHAPTER 2 HL

		Rescaling and normalizing vectors. **AHL 3.11** Vector equation of a line in two and three dimensions. **AHL 3.12** Vector applications to kinematics. Modelling linear motion with constant velocity in two and three dimensions. Motion with variable velocity in two dimensions. **AHL 3.13** Definition and calculation of the scalar product of two vectors. The angle between two vectors; the acute angle between two lines. Definition and calculation of the vector product of two vectors. Geometric interpretation of $\|v \times w\|$. Components of vectors.	
	Topic 2: Functions	**SL 2.2** Concept of a function, domain, range and graph. Function notation, for example. The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$. The concept of a function as a mathematical model. **SL 2.3** The graph of a function. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences. **SL 2.4** Determine key features of graphs. Finding the points of intersection of two curves or lines using technology.	CHAPTER 3 SL
	Topic 1: Number and algebra	**AHL 1.14** Definition of a matrix: the terms element, row, column and order for $m \times n$ matrices. Algebra of matrices: equality; addition; subtraction; multiplication by a scalar for $m \times n$ matrices. Multiplication of matrices. Properties of matrix multiplication: associativity, distributivity and non-commutativity. Identity and zero matrices. Determinants and inverses of $n \times n$ matrices with technology, and by hand for 2×2 matrices. Awareness that a system of linear equations can be written in the form $Ax = b$. Solution of the systems of equations using inverse matrix. **AHL 1.15** Eigenvalues and eigenvectors Characteristic polynomial of 2 × 2 matrices. Diagonalization of 2 × 2 matrices (restricted to the case where there are distinct real eigenvalues). Applications to powers of 2 × 2 matrices.	CHAPTER 3 HL
	Topic 2: Functions	**SL 2.1** Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients m_1 and m_2 Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$.	CHAPTER 4 SL

	Topic 3: Geometry and trigonometry	**AHL 3.7** The definition of a radian and conversion between degrees and radians. Using radians to calculate area of sector, length of arc. **AHL 3.8** he definitions of $cos\theta$ and $sin\theta$ in terms of the unit circle. The Pythagorean identity. Definition of $tan\theta$ as $sin\theta/cos\theta$. Extension of the sine rule to the ambiguous case. Graphical methods of solving trigonometric equations in a finite interval. **AHL 3.9** Geometric transformations of points in two dimensions using matrices: reflections, horizontal and vertical stretches, enlargements, translations and rotations. Compositions of the above transformations. Geometric interpretation of the determinant of a transformation matrix.	CHAPTER 4 HL
	Topic 3: Geometry and trigonometry	**SL 3.1** The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. **SL 3.2** Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule. The cosine rule. Area of a triangle. **SL 3.1** The size of an angle between two intersecting lines or between a line and a plane. **SL 3.3** Applications of right and non-right angled trigonometry, including Pythagoras' theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.	CHAPTER 5 SL
	Topic 2: Functions	**AHL 2.7** Composite functions in context. The notation $(f \circ g)(x) = f(g(x))$. Inverse function, including domain restriction. Finding an inverse function. **AHL 2.8** Transformations of graphs. Translations. Reflections. Vertical stretch. Horizontal stretch. Composite transformations. **AHL 2.9** Exponential models to calculate half-life. Natural logarithmic models. Sinusoidal models. Logistic models. Piecewise models.	CHAPTER 5 HL

	Topic 4: Statistics and probability	**SL 4.1** Concepts of population, sample, random sample, discrete and continuous data. Reliability of data sources and bias in sampling. Interpretation of outliers. Sampling techniques and their effectiveness. **SL 4.2** Presentation of data (discrete and continuous): frequency distributions (tables). Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). Production and understanding of box and whisker diagrams. **SL 4.3** Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. Modal class. Measures of dispersion – IQR. Effect of constant changes on the original data. Quartiles of discrete data. **SL 4.4** Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r. Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.	CHAPTER 6 SL
	Topic 1: Number and algebra	**AHL 1.12** Complex numbers. Cartesian form, real part, imaginary part, conjugate, modulus and argument. Calculate sums, differences, products, quotients, by hand and with technology. Calculating powers of complex numbers, in Cartesian form, with technology. The complex plane. Complex numbers as solutions to quadratic equations. **AHL 1.13** Modulus–argument (polar) form. Exponential form. Conversion between Cartesian, polar and exponential forms, by hand and with technology. Calculate products, quotients and integer powers in polar or exponential forms. Adding sinusoidal functions with the same frequencies but different phase shift angles. Geometric interpretation of complex numbers.	CHAPTER 6 HL
	Topic 4: Statistics and probability	**SL 4.5** Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event A is $P(A) = n(A)/n(U)$. The complementary events A and A' (not A). Expected number of occurrences. **SL 4.6** Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities.	CHAPTER 7 SL

		Combined events. Mutually exclusive events. Conditional probability. Independent events.	
	Topic 3: Geometry and trigonometry	**AHL 3.14** Graph theory: Graphs, vertices, edges, adjacent vertices, adjacent edges. Degree of a vertex. Simple graphs; complete graphs; weighted graphs. Directed graphs; in degree and out degree of a directed graph. Subgraphs; trees. **AHL 3.15** Adjacency matrices. Walks. Number of k-length walks (or less than k-length walks) between two vertices. Weighted adjacency tables. Construction of the transition matrix for a strongly-connected, undirected or directed graph. **AHL 3.16** Tree and cycle algorithms with undirected graphs. Walks, trails, paths, circuits, cycles. Eulerian trails and circuits. Hamiltonian paths and cycles. Minimum spanning tree (MST) graph algorithms. Kruskal's and Prim's algorithms for finding minimum spanning trees. Chinese postman problem and algorithm for solution, to determine the shortest route around a weighted graph with up to four odd vertices, going along each edge at least once. Travelling salesman problem to determine the Hamiltonian cycle of least weight in a weighted complete graph. Nearest neighbour algorithm for determining an upper bound for the travelling salesman problem. Deleted vertex algorithm for determining a lower bound for the travelling salesman problem.	CHAPTER 7 HL
	Topic 4: Statistics and probability	**SL 4.7** Concept of discrete random variables and their probability distributions. Expected value (mean), $E(X)$ for discrete data. Applications. **SL 4.8** Binomial distribution. Mean and variance of the binomial distribution. **SL 4.9** The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation. Normal probability calculations. Inverse normal calculations.	CHAPTER 8 SL
	Topic 4: Statistics and probability	**AHL 4.14** Linear transformation of a single random variable. Expected value of linear combinations of n random variables. Variance of linear combinations of n independent random variables. **AHL 4.15** A linear combination of n independent normal random variables is normally distributed. Central limit theorem. **AHL 4.17**	CHAPTER 8 HL

		Poisson distribution, its mean and variance. Sum of two independent Poisson distributions has a Poisson distribution. **AHL 4.19** Transition matrices. Powers of transition matrices. Regular Markov chains. Initial state probability matrices. Calculation of steady state and long-term probabilities by repeated multiplication of the transition matrix or by solving a system of linear equations.	
	Topic 5: Calculus	**SL 5.1** Introduction to the concept of a limit. Derivative interpreted as gradient function and as rate of change. **SL 5.2** Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$, $f'(x) < 0$. **SL 5.3** Derivative of $f(x) = ax^n$. The derivative of functions of the form $f(x) = ax^n + bx^{n-1}$ … where all exponents are integers. **SL 5.4** Tangents and normals at a given point, and their equations.	CHAPTER 9 SL
	Topic 5: Calculus	**SL 5.5** Introduction to integration as anti-differentiation of functions. Anti-differentiation with a boundary condition to determine the constant term. Definite integrals using technology. Area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.	CHAPTER 10 SL
Year 2	Topic 1: Number and algebra	**SL 1.6** Approximation: decimal places, significant figures. Upper and lower bounds of rounded numbers. Percentage errors. Estimation. **SL 1.7** Amortization and annuities using technology.	CHAPTER 11 SL
	Topic 4: Statistics and probability	**AHL 4.12** Design of valid data collection methods, such as surveys and questionnaires. Selecting relevant variables from many variables. Choosing relevant and appropriate data to analyse. Definition of reliability and validity. Reliability tests. Validity tests. **AHL 4.14** Unbiased estimators of mean and standard deviation. **AHL 4.12** Categorizing numerical data in a χ^2 table and justifying the choice of categorisation. Choosing an appropriate number of degrees of freedom when estimating parameters from data when carrying out the χ^2 goodness of fit test. **AHL 4.13**	CHAPTER 9 HL

		Regression with non-linear functions. Evaluation of least squares regression curves using technology. Sum of square residuals as a measure of fit for a model. The coefficient of determination and its valuation using technology. **AHL 4.16** Confidence intervals for the mean of a normal population. **AHL 4.18** Critical values and critical regions. Test for population mean for normal distribution. Test for proportion using binomial distribution. Test for population mean using Poisson distribution. Use of technology to test the hypothesis that the population product moment correlation coefficient is 0 for bivariate normal distributions. Type I and II errors including calculations of their probabilities.	
	Topic 1: Number and algebra	**SL 1.8** Use technology to solve systems of linear equations in up to 3 variables. Use technology to solve polynomial equations.	CHAPTER 12 SL
	Topic 5: Calculus	**AHL 5.9** The derivatives of elementary functions. The chain rule, product rule and quotient rules. Related rates of change. **AHL 5.10** The second derivative. Use of second derivative test to distinguish between a maximum and a minimum point.	CHAPTER 10 HL
	Topic 2: Functions	**SL 2.5** Linear models. Quadratic models (axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y -axis). Exponential growth and decay models (equation of a horizontal asymptote). Direct/inverse variation (the y-axis as a vertical asymptote). Cubic models. Sinusoidal models. **SL 2.6** Use the modelling process described in the “mathematical modelling” section to create, fit and use the theoretical models. Given a context recognize and choose an appropriate model and possible parameters.Determine a reasonable domain for a model. Find the parameters of a model. Comment on the appropriateness and reasonableness of a model. Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation. Reading, interpreting and making predictions based on the model.	CHAPTER 13 SL
	Topic 5: Calculus	**AHL 5.11** Definite and indefinite integration of elementary functions. Integration by inspection, or substitution. **AHL 5.12** Area of the region enclosed by a curve and the x or y-axes in a given interval. Volumes of revolution.	CHAPTER 11 HL

	Topic 3: Geometry and trigonometry	**SL 3.4** The circle: length of an arc; area of a sector. **SL 3.5** Equations of perpendicular bisectors. **SL 3.6** Voronoi diagrams: sites, vertices, edges, cells. Addition of a site to an existing Voronoi diagram. Nearest neighbour interpolation. Applications of the “toxic waste dump” problem.	CHAPTER 14 SL
	Topic 5: Calculus	**AHL 5.13** Kinematic problems involving displacement s, velocity v and acceleration a.	CHAPTER 12 HL
	Topic 4: Statistics and probability	**SL 4.11** Formulation of null and alternative hypotheses, H_0 and H_1. Significance levels. p-values. Expected and observed frequencies. The χ2 test for independence: contingency tables, degrees of freedom, critical value. The χ2 goodness of fit test. The t-test. Use of the p -value to compare the means of two populations. Using one-tailed and two-tailed tests. **SL 4.10** Spearman’s rank correlation coefficient. Awareness of the appropriateness and limitations of Pearson’s product moment correlation coefficient and Spearman’s rank correlation coefficient, and the effect of outliers on each.	CHAPTER 15 SL
	Topic 5: Calculus	**AHL 5.14** Setting up a model/differential equation from a context. Solving by separation of variables. **AHL 5.15** Slope fields and their diagrams. **AHL 5.16** Euler’s method for finding the approximate solution to first order differential equations. Numerical solution of $dy/dx = f(x, y)$. Numerical solution of the coupled system. **AHL 5.17** Phase portrait for the solutions of coupled differential equations. Qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues. Sketching trajectories and using phase portraits to identify key features such as equilibrium points, stable populations and saddle points. **AHL 5.18** Solutions of second order dif. equations by Euler’s method.	CHAPTER 13 HL
	Topic 5: Calculus	**SL 5.6** Values of x where the gradient of a curve is zero. Solution of $f'(x) = 0$. Local maximum and minimum points.	CHAPTER 16 SL

		SL 5.7 Optimisation problems in context. **SL 5.8** Approximating areas using the trapezoidal rule.	

Assessment instruments to be used	Resources *List the main resources to be used, including information technology if applicable.*
• Self-assessment – using answers in the book • Peer-assessment – discussions with the peers • Questioning • Tests – both individual chapter and key skills tests • EOY and MOCK exams • Assignments – questions completed at home	• The textbook – Mathematics Applications and Interpretation SL and HL • The IB Question bank • Geogebra (www.geogebra.com) • YouTube tutorials (Khans Academy) • GDC – TI CE84 Plus

2. IB internal assessment requirement to be completed during the course

Briefly explain how and when you will work on it. Include the date when you will first introduce the internal assessment requirement to your students, the different stages and when the internal assessment requirement will be due.

By the end of September (Year 2), students are introduced to IA principles. During the rest of the calendar year, students are shown sample IAs. Students also brainstorm ideas that match their interests to be explored in their own IAs. In January, students submit their IA topic selections. By the end of Faburary, students can submit their first drafts and get feedback from their subject teacher. By the middle of March they submit their final drafts of the IA.

3. Links to TOK

You are expected to explore links between the topics of your subject and TOK. As an example of how you would do this, choose one topic from your course outline that would allow your students to make links with TOK. Describe how you would plan the lesson.

Topic	Link with TOK (including description of lesson plan)
Amortization and annuities using technology.	How have technological advances affected the nature and practice of mathematics? • Introduction of the problem – saving regular amounts over a given time span and with fixed interest rate. • Attempt to solve it using a scientific calculator – considering the issue of accuracy and effectiveness of the method • Attempt to solve it using the spreadsheet – showing appreciation for the use of repeated formulae connecting the cells • Attempt to solve using the TVM application using a GDC – considering the shift in the need to understand the underlying maths concept to understanding the app variables.

4. Approaches to learning

Every IB course should contribute to the development of students' approaches to learning skills. As an example of how you would do this, choose one topic from your outline that would allow your students to specifically develop one or more of these skill categories (thinking, communication, social, self-management or research).

Topic	Contribution to the development of students' approaches to learning skills (including one or more skill category)
Applies to most topics.	Self-management: most exercises contain sets of repetitive questions to allow practising the key skills introduced in the corresponding chapters. They are followed by exam style questions. Students must self-evaluate their knowledge and understanding to decide where to focus their efforts. Their decision needs to take into consideration the time they can afford to spend doing maths.

5. International mindedness

Every IB course should contribute to the development of international-mindedness in students. As an example of how you would do this, choose one topic from your outline that would allow your students to analyse it from different cultural perspectives. Briefly explain the reason for your choice and what resources you will use to achieve this goal.

Topic	Contribution to the development of international mindedness (including resources you will use)
Applies to most topics.	Students need to appreciate the differences between the use of certain symbols in the IB maths and the standard Czech curriculum (decimal point vs decimal comma, multiplication and division symbols, …).

6. Development of the IB learner profile

Through the course it is also expected that students will develop the attributes of the IB learner profile. As an example of how you would do this, choose one topic from your course outline and explain how the contents and related skills would pursue the development of any attribute(s) of the IB learner profile that you will identify.

Topic	Contribution to the development of the attribute(s) of the IB learner profile
Binomial distribution	Learning about the Binomial distribution fits the IB program's definition of being 'knowledgeable' in several ways: Conceptual Understanding: The Binomial distribution is a core concept in probability theory, representing situations where there are two possible outcomes (e.g., success/failure). Understanding this distribution involves grasping how probabilities are distributed across trials, which strengthens one's mathematical reasoning and critical thinking skills. Exploring Knowledge Across Disciplines: - Mathematics and Statistics: The Binomial distribution is fundamental in these fields. It is used to model real-life scenarios, from simple coin flips to more complex events like the likelihood of success in a series of experiments. - Science and Engineering: Many applications in science (e.g., genetics, physics, and medicine) rely on binomial models to predict outcomes or measure risks. - Economics and Social Sciences: In fields like economics, business, and social studies, the Binomial distribution helps model decision-making under uncertainty, such as in risk analysis or forecasting. Engagement with Local and Global Issues: - Locally, binomial models can help in decision-making processes, like predicting the success of new business ventures or public health interventions. - Globally, the Binomial distribution can be applied to understanding broader phenomena, such as the spread of diseases, election outcomes, or global market trends. It helps in analysing data and making predictions on global issues such as sustainability, healthcare, and economics.

7. Resources

Describe the resources that you and your student will have to support the subject. Indicate whether they are sufficient in terms of quality, quantity and variety. Briefly describe what plans are in place if changes are needed.

The specific resources are mentioned above. It is felt that they are sufficient in terms of quality, quantity and variety. The nature of the subject requires regular practice from the students which these resources fully support.