Diploma Programme subject outline–Group 5: mathematics			
School name	**Gymnazium a SOS Rokycany**	**School code**	**061768**
Name of the DP subject *(indicate language)*	**Mathematics: analysis and approaches SL and HL (English)**		
Level *(indicate with X)*	Higher [X] Standard completed in two years [X] Standard completed in one year * [ ]		
Name of the teacher who completed this outline	**Tomas Trhlik**	**Date of IB training**	**2022**
Date when outline was completed	**Updated 09/2024**	**Name of workshop** *(indicate name of subject and workshop category)*	**Maths AI, Cat 1**

* All Diploma Programme courses are designed as two-year learning experiences. However, up to two standard level subjects, excluding languages ab initio and pilot subjects, can be completed in one year, according to conditions established in the *Handbook of procedures for the Diploma Programme*.

1. Course outline

– Use the following table to organize the topics to be taught in the course. If you need to include topics that cover other requirements you have to teach (for example, national syllabus), make sure that you do so in an integrated way, but also differentiate them using italics. Add as many rows as you need.

– This document should not be a day-by-day accounting of each unit. It is an outline showing how you will distribute the topics and the time to ensure that students are prepared to comply with the requirements of the subject.

– This outline should show how you will develop the teaching of the subject. It should reflect the individual nature of the course in your classroom and should not just be a “copy and paste” from the subject guide.

– If you will teach both higher and standard level, make sure that this is clearly identified in your outline.

	Topic/unit	Contents	Allocated time One class is 45 min. In one week there are 3 SL/HL classes + 1 HL class.
Year 1	Topic 4: Statistics and probability	**SL 4.5** Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event A is $P(A) = n(A)/n(U)$. The complementary events A and A' (not A). Expected number of occurrences. **SL 4.6** Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities. Combined events. Mutually exclusive events. Conditional probability. Independent events.	CHAPTER 7 SL
	Topic 4: Statistics and probability	**SL 4.7** Concept of discrete random variables and their probability distributions. Expected value (mean), $E(X)$ for discrete data. Applications. **SL 4.8** Binomial distribution. Mean and variance of the binomial distribution. **SL 4.9** The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation. Normal probability calculations. Inverse normal calculations.	CHAPTER 8 SL
	Topic 1: Number and algebra	**AHL 1.10** Counting principles, including permutations and combinations.	CHAPTER 1 HL
	Topic 1: Number and algebra	**AHL 1.10** Extension of the binomial theorem to fractional and negative indices. **AHL 1.11** Partial fractions. **AHL 1.16** Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.	CHAPTER 2 HL

	Topic 2: Functions	**SL 2.1** Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients m_1 and m_2 Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$.	CHAPTER 4 SL
	Topic 3: Geometry and trigonometry	**SL 3.1** The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. **SL 3.2** Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule. The cosine rule. Area of a triangle. **SL 3.1** The size of an angle between two intersecting lines or between a line and a plane. **SL 3.3** Applications of right and non-right angled trigonometry, including Pythagoras' theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.	CHAPTER 5 SL
	Topic 3: Geometry and trigonometry	**AHL 3.9** Definition of the reciprocal trigonometric ratios. Pythagorean identities. The inverse trig functions, their domains, ranges and their graphs. **AHL 3.10** Compound angle identities. Double angle identity for tan. **AHL 3.11** Relationships between trigonometric functions and the symmetry properties of their graphs.	CHAPTER 3 HL
	Topic 3: Geometry and trigonometry	**AHL 3.12** Concept of a vector; position vectors; displacement vectors. Representation of vectors using directed line segments. Base vectors. Components of a vector. Algebraic and geometric approaches to the following: • the sum and difference of two vectors • the zero vector 0, the vector $-v$ • multiplication by a scalar, kv, parallel vector • magnitude of a vector, $\|v\|$, unit vectors, v $\|v\|$ • position vectors $OA \rightarrow = a$, $OB \rightarrow = b$ • displacement vector $AB \rightarrow = b - a$ Proofs of geometrical properties using vectors. **AHL 3.13** The definition of the scalar product of two vectors. The angle between two vectors. Perpendicular vectors; parallel vectors.	CHAPTER 8 HL

		AHL 3.14 Vector equation of a line in two and three dimensions. The angle between two lines. Simple applications to kinematics. **AHL 3.15** Coincident, parallel, intersecting and skew lines, distinguishing between these cases. Points of intersection. **AHL 3.16** The definition of the vector product of two vectors. Properties of the vector product and its geometric interpretation.	
	Topic 1: Number and algebra	**SL 1.5** Laws of exponents with integer exponents. **SL 1.1** Operations with numbers in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer **SL 1.5** Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.	CHAPTER 1 SL
	Topic 1: Number and algebra	**SL 1.2** Arithmetic sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. Applications. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life. **SL 1.3** Geometric sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for the sums of geometric sequences. Applications. **SL 1.4** Financial applications of geometric sequences and series.	CHAPTER 2 SL
	Topic 2: Functions	**AHL 2.12** Polynomial functions, their graphs and equations; zeros, roots and factors. The factor and remainder theorems. Sum and product of the roots of polynomial equations.	CHAPTER 6 HL
	Topic 2: Functions	**SL 2.2** Concept of a function, domain, range and graph. Function notation, for example. The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$. The concept of a function as a mathematical model. **SL 2.3** The graph of a function. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.	CHAPTER 3 SL

		SL 2.4 Determine key features of graphs. Finding the points of intersection of two curves or lines using technology.	
	Topic 4: Statistics and probability	**SL 4.1** Concepts of population, sample, random sample, discrete and continuous data. Reliability of data sources and bias in sampling. Interpretation of outliers. Sampling techniques and their effectiveness. **SL 4.2** Presentation of data (discrete and continuous): frequency distributions (tables). Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). Production and understanding of box and whisker diagrams. **SL 4.3** Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. Modal class. Measures of dispersion – IQR. Effect of constant changes on the original data. Quartiles of discrete data. **SL 4.4** Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r. Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.	CHAPTER 6 SL
	Topic 1: Number and algebra	**AHL 1.12** Complex numbers. Cartesian form, the terms real part, imaginary part, conjugate, modulus and argument. Complex plane. **AHL 1.13** Modulus–argument (polar) form. Euler form. Sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation. **AHL 1.14** Complex conjugate roots of quadratic and polynomial equations with real coefficients. De Moivre's theorem and its extension to rational exponents. Powers and roots of complex numbers.	CHAPTER 4 HL
	Topic 1: Number and algebra	**AHL 1.15** Proof by mathematical induction. Proof by contradiction. Use of a counterexample to show that a statement is not always true.	CHAPTER 5 HL

	Topic 5: Calculus	**SL 5.1** Introduction to the concept of a limit. Derivative interpreted as gradient function and as rate of change. **SL 5.2** Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$, $f'(x) < 0$. **SL 5.3** Derivative of $f(x) = ax^n$. The derivative of functions of the form $f(x) = ax^n + bx^{n-1}$ ... where all exponents are integers. **SL 5.4** Tangents and normals at a given point, and their equations.	CHAPTER 9 SL
	Topic 5: Calculus	**SL 5.5** Introduction to integration as anti-differentiation of functions. Anti-differentiation with a boundary condition to determine the constant term. Definite integrals using technology. Area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.	CHAPTER 10 SL
	Topic 1: Number and algebra	**SL 1.6** Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof. The symbols and notation for equality and identity.	CHAPTER 11 SL
	Topic 1: Number and algebra	**SL 1.7** Laws of exponents with rational exponents. Laws of logarithms. Change of base of a logarithm. Solving exponential equations, including using logarithms.	CHAPTER 12 SL
	Topic 1: Number and algebra	**SL 1.8** Sum of infinite convergent geometric sequences. **SL 1.9** The binomial theorem. Pascal's triangle and nCr.	CHAPTER 13 SL
	Topic 2: Functions	**SL 2.5** Composite functions. Identity function. Finding the inverse function.	CHAPTER 14 SL
Year 2	Topic 2: Functions	**SL 2.6** The quadratic function. Graph, intercepts, axis of symmetry, vertex. **SL 2.7** Solution of quadratic equations and inequalities. The quadratic formula. The discriminant and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.	CHAPTER 15 SL
	Topic 2: Functions	**AHL 2.13** Rational functions. **AHL 2.14** Odd and even functions. Finding the inverse function including domain restriction.	CHAPTER 7 HL

		Self-inverse functions. **AHL 2.15** Solutions of $g(x) \geq f(x)$, both graphically and analytically. **AHL 2.16** Functions with modulus. Solution of modulus equations and inequalities.	
	Topic 2: Functions	**SL 2.11** Transformations of graphs. Translations. Reflections (in both axes). Vertical stretch with scale factor p. Horizontal stretch with scale factor $1/q$. Composite transformations. **SL 2.8** The reciprocal function, its graph and self-inverse nature. Rational functions and their graphs. Equations of vertical and horizontal asymptotes. **SL2.9** Exponential functions and their graphs. Logarithmic functions and their graphs.	CHAPTER 16 SL
	Topic 2: Functions	**SL 2.10** Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.	CHAPTER 17 SL
	Topic 3: Geometry and trigonometry	**SL.3.4** The circle: radian measure of angles; length of an arc; area of a sector. **SL 3.5** Definition of $cos\theta$, $sin\theta$ in terms of the unit circle. Definition of $tan\theta$. Exact values of trigonometric ratios and their multiples. Extension of the sine rule to the ambiguous case. **SL 3.6** The Pythagorean identity. Double angle identities for sine and cosine. The relationship between trigonometric ratios. **SL 3.7** The circular functions $sinx$, $cosx$, and $tanx$; amplitude, their periodic nature, and their graphs. Composite functions of the form $f(x) = asin(b(x + c)) + d$. Transformations. Real-life contexts. **SL 3.8** Solving trigonometric equations in a finite interval, both graphically and analytically. Equations leading to quadratic equations in $sinx$, $cosx$ or $tanx$.	CHAPTER 18 SL

	Topic 4: Statistics and probability	**SL 4.10** Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$. **SL 4.11** Conditional probability. **SL 4.12** Standardization of normal variables (z-values). Inverse normal calculations where mean or standard deviation are unknown.	CHAPTER 19 SL
	Topic 5: Calculus	**SL 5.6** Derivatives of elementary functions. Differentiation of a sum and a multiple of these functions. The chain rule for composite functions. The product and quotient rules. **SL 5.7** The second derivative. Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''. **SL 5.8** Local maximum and minimum points. Testing for maximum and minimum. Optimisation. Points of inflexion with zero and non-zero gradients.	CHAPTER 20 SL
	Topic 5: Calculus	**SL 5.10** Indefinite integral of elementary functions. The composites of any of these with the linear function $ax + b$. Integration by inspection (reverse chain rule) or by substitution. **SL 5.11** Definite integrals, including analytical approach. Areas of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x)$ can be positive or negative, without the use of technology. Areas between curves. **SL 5.9** Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled.	CHAPTER 21 SL
	Topic 4: Statistics and probability	**AHL 4.13** Use of Bayes' theorem for a maximum of three events. **AHL 4.14** Variance of a discrete random variable. Continuous random variables and their probability density functions. Mode and median of continuous random variables. Mean, variance and standard deviation of both discrete and continuous random variables. The effect of linear transformations of X.	CHAPTER 9 HL
	Topic 5: Calculus	**AHL 5.12** Informal understanding of continuity and differentiability of a function at a point. Understanding of limits (convergence and divergence). Definition of derivative from first principles. Higher derivatives. **AHL 5.13**	CHAPTER 10 HL

		The evaluation of limits using l'Hôpital's rule or the Maclaurin series. Repeated use of l'Hôpital's rule. **AHL 5.14** Implicit differentiation. Related rates of change. Optimisation problems. **AHL 5.15** Derivatives of other functions. Indefinite integrals of the derivatives of any of the above functions. The composites of any of these with a linear function. Use of partial fractions to rearrange the integrand. **AHL 5.16** Integration by substitution. Integration by parts. Repeated integration by parts. **AHL 5.17** Area of the region enclosed by a curve and the y-axis in a given interval. Volumes of revolution about the x-axis or y-axis.	
	Topic 5: Calculus	**AHL 5.18** First order differential equations. Numerical solution of $dy/dx = f(x, y)$ using Euler's method. Variables separable. Homogeneous differential equation. Using the integrating factor. **AHL 5.19** Maclaurin series. Use of simple substitution, products, integration and differentiation to obtain other series. Maclaurin series developed from differential equations.	CHAPTER 11 HL

Assessment instruments to be used	Resources *List the main resources to be used, including information technology if applicable.*
• Self-assessment – using answers in the book • Peer-assessment – discussions with the peers • Questioning • Tests – both individual chapter and key skills tests • EOY and MOCK exams • Assignments – questions completed at home • Presentations	• The textbook – Mathematics Applications and Interpretation SL and HL • The IB Question bank • Geogebra (www.geogebra.com) • YouTube tutorials (Khans Academy) • GDC – TI 84 Plus CE

2. IB internal assessment requirement to be completed during the course

Briefly explain how and when you will work on it. Include the date when you will first introduce the internal assessment requirement to your students, the different stages and when the internal assessment requirement will be due.

By the end of September (Year 2), students are introduced to IA principles. During the rest of the calendar year, students are shown sample IAs. Students also brainstorm ideas that match their interests to be explored in their own IAs. In January, students submit their IA topic selections. By the end of Faburary, students can submit their first drafts and get feedback from their subject teacher. By the middle of March they submit their final drafts of the IA.

3. Links to TOK

You are expected to explore links between the topics of your subject and TOK. As an example of how you would do this, choose one topic from your course outline that would allow your students to make links with TOK. Describe how you would plan the lesson.

Topic	Link with TOK (including description of lesson plan)
Probability	Is it possible to have knowledge of the future? • Introduction of the problem – simple probability of tossing a fair coin. • Considering expectations based on probability VS reality. • Simulation of the problem using spreadsheet – distant future – what is to happen? • Is it still possible to have a large series of HEADS?

4. Approaches to learning

Every IB course should contribute to the development of students' approaches to learning skills. As an example of how you would do this, choose one topic from your outline that would allow your students to specifically develop one or more of these skill categories (thinking, communication, social, self-management or research).

Topic	Contribution to the development of students' approaches to learning skills (including one or more skill category)
Applies to most topics.	Self-management: most exercises contain sets of repetitive questions to allow practising the key skills introduced in the corresponding chapters. They are followed by exam style questions. Students must self-evaluate their knowledge and understanding to decide where to focus their efforts. Their decision needs to take into consideration the time they can afford to spend doing maths.

5. International mindedness

Every IB course should contribute to the development of international-mindedness in students. As an example of how you would do this, choose one topic from your outline that would allow your students to analyse it from different cultural perspectives. Briefly explain the reason for your choice and what resources you will use to achieve this goal.

Topic	Contribution to the development of international mindedness (including resources you will use)
Applies to most topics.	Students need to appreciate the differences between the use of certain symbols in the IB maths and the standard Czech curriculum (decimal point vs decimal comma, multiplication and division symbols, …).

6. Development of the IB learner profile

Through the course it is also expected that students will develop the attributes of the IB learner profile. As an example of how you would do this, choose one topic from your course outline and explain how the contents and related skills would pursue the development of any attribute(s) of the IB learner profile that you will identify.

Topic	Contribution to the development of the attribute(s) of the IB learner profile
Binomial distribution	Learning about the Binomial distribution fits the IB program's definition of being 'knowledgeable' in several ways: Conceptual Understanding: The Binomial distribution is a core concept in probability theory, representing situations where there are two possible outcomes (e.g., success/failure). Understanding this distribution involves grasping how probabilities are distributed across trials, which strengthens one's mathematical reasoning and critical thinking skills. Exploring Knowledge Across Disciplines: - Mathematics and Statistics: The Binomial distribution is fundamental in these fields. It is used to model real-life scenarios, from simple coin flips to more complex events like the likelihood of success in a series of experiments. - Science and Engineering: Many applications in science (e.g., genetics, physics, and medicine) rely on binomial models to predict outcomes or measure risks. - Economics and Social Sciences: In fields like economics, business, and social studies, the Binomial distribution helps model decision-making under uncertainty, such as in risk analysis or forecasting. Engagement with Local and Global Issues: - Locally, binomial models can help in decision-making processes, like predicting the success of new business ventures or public health interventions. - Globally, the Binomial distribution can be applied to understanding broader phenomena, such as the spread of diseases, election outcomes, or global market trends. It helps in analysing data and making predictions on global issues such as sustainability, healthcare, and economics.

7. Resources

Describe the resources that you and your student will have to support the subject. Indicate whether they are sufficient in terms of quality, quantity and variety. Briefly describe what plans are in place if changes are needed.

The specific resources are mentioned above. It is felt that they are sufficient in terms of quality, quantity and variety. The nature of the subject requires regular practice from the students which these resources fully support.