Diploma Programme subject outline–Group 5: mathematics			
School name	**Gymnazium a SOS Rokycany**	**School code**	**061768**
Name of the DP subject *(indicate language)*	**Mathematics: analysis and approaches**		
Level *(indicate with X)*	Higher [ ] Standard completed in two years [x] Standard completed in one year * [ ]		
Name of the teacher who completed this outline	Lenka Likeova	**Date of IB training**	June 2021
Date when outline was completed	05/2021	**Name of workshop** *(indicate name of subject and workshop category)*	Mathematics: Analysis and approaches (Cat. 1)

* All Diploma Programme courses are designed as two-year learning experiences. However, up to two standard level subjects, excluding languages ab initio and pilot subjects, can be completed in one year, according to conditions established in the *Handbook of procedures for the Diploma Programme*.

1. Course outline

- Use the following table to organize the topics to be taught in the course. If you need to include topics that cover other requirements you have to teach (for example, national syllabus), make sure that you do so in an integrated way, but also differentiate them using italics. Add as many rows as you need.
- This document should not be a day-by-day accounting of each unit. It is an outline showing how you will distribute the topics and the time to ensure that students are prepared to comply with the requirements of the subject.
- This outline should show how you will develop the teaching of the subject. It should reflect the individual nature of the course in your classroom and should not just be a “copy and paste” from the subject guide.
- If you will teach both higher and standard level, make sure that this is clearly identified in your outline.

	Topic/unit (as identified in the IB subject guide) *State the topics/units in the order you are planning to teach them.*	Contents	Allocated time One class is 45 minutes. In one week there are 4,3 classes. (year 1 – 4 classes, year 2 – 3 classes)	Assessment instruments to be used	Resources *List the main resources to be used, including information technology if applicable.*
Year 1	**Unit 0:** Operations with numbers and expressions, simple deductive proofs	Review of prior learning topics: Prime numbers, LCM, GDC, Polynomials, Rational expressions, Algebraic expressions, Solution of quadratic equations and inequalities	**6 hours** (8 classes)	**Formative assessment:** Reasoning – when doing simple deductive proofs (SL 1.6) **Summative assessment:** At the end of unit 2, covering units 0, 1, 2 (written exam)	IB course books, Khan Academy Math, YouTube videos, online articles, SW GeoGebra, SW Graph, GDC (see end of this document)
		Scientific notation of number and operations in that form (SL 1.1) **Number and Algebra**			
		Simple deductive proof, symbols and notation (SL 1.6) **Number and Algebra**			
	Unit 1: Functions	Concept of function, domain, range (SL 2.2) Graph of a function, key features of graphs, finding the points of intersection of two curves or lines (SL 2.3, SL 2.4) Different forms of the equation of a straight line, gradient, intercepts (SL 2.1) **Functions**	**10,5 hours** (14 classes)		
		Reciprocal function, Equations vertical and horizontal asymptotes (SL 2.8) Transformations of graphs (SL 2.11) **Functions**			
		Composite functions, Identity function, Inverse function, (SL 2.2, SL 2.5) **Functions**			
	Unit 2: Quadratic function	All forms of quadratic function, graph (SL 2.6) Solution of quadratic equations and inequalities, the quadratic formula (SL 2.7) The discriminant (SL 2.7)	**6 hours** (8 classes)		

		Functions			
		Finding point of intersection of two curves (SL 2.4) **Functions**			
	Unit 3: Exponential and logarithmic function	Laws of exponents with integer (SL 1.5) Laws of rational exponents (SL 1.7) Exponential functions and their graphs (SL 2.9) **Functions**	**Number and algebra** **6 hours** (8 classes)	**Exploration:** Use of the logarithmic scale: music scale, sound intensity level, pH scale, Richter scale, Shannon-Weaver Index etc.	
		Introduction to logarithms with base 10 and *e* (SL 1.5) Laws of logarithms, change of base of logarithm (SL 1.7) **Number and Algebra** Logarithmic functions and their graphs (SL 2.9) **Functions**	**Functions** **9 hours** (12 classes)		
		Numerical evaluation of logarithms using technology (SL 1.5) Solving exponential equations, including using logarithms. (SL 1.7) **Number and algebra, Functions**			
	Unit 4: Polynomial and rational functions	Rational functions (SL 2.8) **Functions**	**4,5 hours** (6 classes)		
	Unit 5: Functions and technology	Use of technology to solve a variety of equations (SL 2.10) **Functions**	**4,5 hours** (6 classes)	**Summative assessment:** At the end of unit 5, covering units 1, 2, 3, 4, 5 (written exam)	
	Unit 6: Sequences and series	Arithmetic sequences and series Use of the formulae *n*-th term, sum of *n* terms Use of sigma notation for sums of arithmetic sequences Applications, interpretation and prediction (SL 1.2) **Number and Algebra**	**15 hours** (20 classes)	**Formative assessment:** Solving practical examples with using series **Summative assessment:** At the end of	
		Geometric sequences and series Use of the formulae for *n*-th term, sum of *n* terms Use of sigma notation´ Applications (SL 1.3)			

		Number and Algebra Sum of infinite convergent geometric sequences (SL 1.8) Financial applications of geometric sequences and series: compound interest, annual depreciation (SL 1.4) **Number and Algebra**		unit 6 (written exam)	
	Unit 7: Trigonometry	Definition of $\cos\theta$, $\sin\theta$ in terms of the unit circle (SL 3.5) Definition of $\tan\theta$, reciprocal trigonometric ratios (SL 3.5) The Pythagorean identity (SL 3.6) **Geometry and trigonometry** Double angle identities for sine and cosine (SL 3.6) The relationship between trigonometric ratios (SL 3.6) Exact values of trigonometric ratios (SL 3.5) **Geometry and trigonometry** The circular functions *sin x, cos x, tan x* (SL 3.7) Amplitude, periodic nature, graphs (SL 3.7) Composite functions of the form, transformations, real-life contexts (SL 3.7) **Geometry and trigonometry** Solving trigonometric equations (SL 3.8) Equations leading to quadratic equations in *sin x, cos x or tan x* (SL 3.8) **Geometry and trigonometry**	**18 hours** (24 classes)	**Formative assessment:** Using GeoGebra, Desmos or GDC to investigate trigonometric functions and their inverse functions	
	Unit 8: Geometry in 2D and 3D	Use of sin, cos and tan ratios to find the sides and angles of right-angled triangles (SL 3.2) The sine and cosine rule. (SL 3.2) Extension of the sine and cosine rule to the ambiguous case (SL 3.5) Area of triangle (SL 3.2) Applications of right and non-right-angled trigonometry (SL 3.3) The circle; radian measure of angles; length of an arc; area of a sector (SL 3.4) **Geometry and trigonometry** 3D shapes, volumes, surface areas (pyramid, cone, …) (SL 3.1) Distance between two points in 3D, (SL 3.1)	**12 hours** (16 classes)	**Summative assessment:** At the end of unit 8, covering units 7, 8 (written exam)	

		Geometry and trigonometry			
	Unit 10: Counting principles	Counting principles, binomial theorem (SL 1.9) Use of Pascal's triangle (SL 1.9) **Number and Algebra**	**4,5 hours** (6 classes)		
Year 2	**Unit 11**: Statistics	Simple statistics from discrete data (mean, mode, median, range) Concept of population, (random) sample, discrete and continuous data (SL 4.1) Reliability of data sources, bias in sampling (SL 4.1) Interpretation of outliers (SL 4.1) Presentation of data (discrete and continuous) (SL 4.2) Histograms, cumulative frequency, cumulative frequency graphs, Production and understanding of box and whisker diagrams (SL 4.2) Measures of central tendency Estimation of mean from grouped data, modal class, measures of dispersion (IQR, standard deviation and variance), quartiles, percentiles, effect of constant changes (SL 4.3) **Statistics and probability**	**9 hours** (12 classes)	**Formative assessment:** Evaluating statements about probability (Equally likely events, Randomness, Sample size) **Formative assessment:** Statistical investigation	IB course books, Khan Academy Math, YouTube videos, online articles, https://www.symbolab.com/, SW GeoGebra, SW Graph, GDC (see end of this document)
	Unit 12: Correlation	Linear correlation of bivariate data (SL 4.4) Pearson's correlation coefficient, scatter diagrams, lines of best fit (SL 4.4) Use of the equation of the regression line for prediction purposes, interpret the meaning of the parameters, *a and b*, in a linear regression *y=ax+b* (SL 4.4) Equation of the regression line of *x* on *y*, use of the equation for prediction purposes (SL 4.10) **Statistics and probability**	**9 hours** (12 classes)		
	Unit 13: Probability and Probability distribution	Concepts: trial, outcome, equally likely outcomes, relative frequency, sample space, event, the probability of an event, the complementary events, expected number. of occurrences (SL 4.5) Use of Venn diagrams, tree diagrams, sample space	**9 hours** (12 classes)	**Exploration:** Volume optimization of a cube or a cuboid or a box	

		diagrams and tables of outcomes to calculate probabilities (SL 4.6) Combined events, mutually exclusive events, conditional probabilities, independent events, formal definition and use of the formulae (SL 4.6, SL 4.11) Concept of discrete random variables and their probability distributions. Applications Binomial distribution (SL 4.7) Mean and variance of the binomial distribution (SL 4.8) The normal distribution and curve, properties of the normal distribution, diagrammatic representation (SL 4.9) Standardization of normal variables, inverse normal calculations where mean and standard deviation are unknown (SL 4.12) **Statistics and probability**		**Summative assessment:** At the end of unit 13, covering units 10–13 (written exam) **Summative assessment:** At the end of unit 15, covering units 14–15 (written exam)	
	Unit 14: Derivatives	Limits, definition of derivative, derivative interpreted as gradient function and as rate of change (SL 5.1) Derivative functions, derivative of polynomials (SL 5.3) Tangents and normals (SL 5.4) Differentiation of a sum (SL 5.6) Differentiation of a quotient, the chain rule (SL 5.6) Derivatives of trig functions, logarithms (SL 5.6) **Calculus**	**15 hours** (20 classes)		
		Increasing and decreasing functions, graphical interpretation of +/- derivative (SL 5.2) The second derivative, graphical behaviour of functions, including relationship between the graphs of f, f', f'' (SL 5.7) Local max and min, testing for maximum and minimum (SL 5.8) Optimization, points of inflexion with zero and non-zero gradients (SL 5.8) **Calculus**			
	Unit 15: Integrals	Definite integrals using technology. (SL 5.5) Integration as anti-differentiation, anti-differentiation with a boundary condition to determine the constant term (SL 5.5) Indefinite integral, composites with linear function,	**15 hours** (20 classes)		

		substitution (SL 5.10) **Calculus**			
		Definite integrals, including analytical approach (SL 5.11) Areas of a region enclosed by a curve and the *x*-axis, without the use of technology. Areas between curves (SL 5.11) **Calculus**			
	Toolkit and Mathematical exploration		**30 hours** (45 classes)		

2. IB internal assessment requirement to be completed during the course

Briefly explain how and when you will work on it. Include the date when you will first introduce the internal assessment requirement to your students, the different stages and when the internal assessment requirement will be due.

General information
Internal assessment is an integral part of the course and is compulsory for both SL and HL students. It enables students to demonstrate the application of their skills and knowledge and to pursue their personal interests, without the time limitations and other constraints that are associated with written examinations. The internal assessment requirements at SL and at HL are the same.

Scheduling (Year 2)
By the end of September (year 2), students are introduced to IA principles. In the following 9 weeks (first half of December), students are shown the sample IAs. Then students brainstorm ideas that match their interests and that they can imagine working on in their own IAs. At the end of December, students submit one to three selected topics and in the second week of January, they finally choose their topic. After this deadline, they are not allowed to change their choice of topic. By the beginning of March, students will submit their first draft. By the end of April, they will complete their final draft of the IAs.

Assessment criteria
During the process of evaluation, Presentation (4 marks), Mathematical communication (4 marks), Personal engagement (3 marks), Reflection (3 Marks), Use of mathematics (6 marks), will be considered.
For more detail see the Mathematics: analysis and approaches guide.

3. Links to TOK

You are expected to explore links between the topics of your subject and TOK. As an example of how you would do this, choose one topic from your course outline that would allow your students to make links with TOK. Describe how you would plan the lesson.

Topic	Link with TOK (including description of lesson plan)
Topic 1. Number and algebra 1.2 Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life	When a model is not perfectly arithmetic in real life, it can still be used to model the process. To understand this, students will be asked to model an arithmetic sequence on a specific practical example and learn about the Fibonacci sequence. Previous to this class, students will collect and bring pine cones and snail shells. A class discussion: Is it possible to model a process when the model is not perfectly arithmetic in real life? After the discussion, students will be divided into two groups to deal with two following problems:

	Group n. 1 Students will be asked to use data of the grammar school students from previous years to model the data as an arithmetic sequence with the goal to predict the development of graduates in 2025. Students will also discuss the following questions: What differential did you choose for modelling and why? Would the data differ if you chose a different differential? How can you know if the modelling makes sense? **Group n. 2** Students will be asked to analyze the sequence f(n+2) = f(n) + f(n+1) and discuss the following questions: What is the sequence if the first member is 0? Can this sequence be found in the plant kingdom? Can this sequence be found in the animal kingdom? Do cabbage layers really show the Fibonacci sequence? Notice how the Fibonacci sequence is manifested in a pine cone, sunflower, snail shell etc. Students will share their results with the rest of the class and discuss the following TOK questions: Is it possible to simplify the model in this way? If the model is not perfect, does that mean it is not useful? Which is better - a precise answer unlikely to be perfectly correct or an imprecise answer which is likely to include the correct answer? Is it necessary to know the history of Fibonacci's sequence (golden ratio) to understand it fully? How can we know that mathematical fact is true? What areas of human life are influenced by mathematics? How can we know that we can trust predictions based on mathematical analyses? Are there any connections between mathematics and art?

4. Approaches to learning

Every IB course should contribute to the development of students' approaches to learning skills. As an example of how you would do this, choose one topic from your outline that would allow your students to specifically develop one or more of these skill categories (thinking, communication, social, self-management or research).

Topic	Contribution to the development of students' approaches to learning skills (including one or more skill category)
Topic 2 Calculus 5.11 Defining integrals, including analytical approach	Students will be given examples of two functions (e.g. $y=(x+1)^2$ and $y=1-x$), and will use Graph software to indicate the content of the resulting shape. *Thinking skills* - students think about possible approaches to solving the problem at hand and consider their pros and cons. *Research skills* - students use Graph and GDC software to improve their abilities to work with modern technologies and be able to research, compare and process data using various IT *Communication skills* - Students improve these skills by passing the results to other classmates and by discussing the process of solving different functions.

5. International mindedness

Every IB course should contribute to the development of international-mindedness in students. As an example of how you would do this, choose one topic from your outline that would allow your students to analyse it from different cultural perspectives. Briefly explain the reason for your choice and what resources you will use to achieve this goal.

Topic	Contribution to the development of international mindedness (including resources you will use)
Topic 2 Functions 2.9. Exponential functions, logarithmic functions	Is Euler's number really named after the mathematician who discovered it? Leonhard Euler, John Napier, Jacob Bernoulli - students study information about the mathematicians and present a concise comparison. Although Leonhard Euler was Swiss, his portrait appeared on Russian and German stamps, students find out and explain the circumstances. The aim of the lesson is to prove that Mathematics is an international language and mathematicians from around the world can communicate effectively within their field. An important outcome to learn is that Mathematics can transcend politics, religion and nationality.

6. Development of the IB learner profile

Through the course it is also expected that students will develop the attributes of the IB learner profile. As an example of how you would do this, choose one topic from your course outline and explain how the contents and related skills would pursue the development of any attribute(s) of the IB learner profile that you will identify.

Topic	Contribution to the development of the attribute(s) of the IB learner profile
Topic 4 Statistics and probability 4.2 Presentation of data (discrete and continuous) frequency distributions. Histograms.	Students present data (sciences, individuals and societies) and they discuss the different formulae for the same statistical measure. By doing this, the students develop these attributes: **Knowledgeable:** Students use math to get data. **Communicators:** Students use technical terminology to inform classmates about statistics. **Thinkers:** Students judge information critically when doing research and solving problems. **Reflective:** Students think about ways to improve their learning. **Risk-takers:** Students develop procedures and skills to solve even new problems. **Inquirers:** Students research various ways to present the data effectively.

7. Resources

Describe the resources that you and your student will have to support the subject. Indicate whether they are sufficient in terms of quality, quantity and variety. Briefly describe what plans are in place if changes are needed.

The school's classrooms are all equipped with computers, multimedia projectors, touch boards, speakers, and high-speed Wi-Fi. There is a computer lab and a well-equipped library with several multimedia and VR stations accessible to students. The school has purchased teacher resource materials for every subject including textbooks, subject guides and teaching methodology material. There is also a virtual link to the library of Western Bohemia University in Pilsen which enables students and teachers to use a wide variety of resources, magazine articles, fiction and non-fiction literature, etc.
There are also printing and scanning stations available to students and teachers enabling them to work with and create various teaching and learning materials.
Overall, the amount and quality of available resources is sufficient to give effective support to the Mathematics: analysis and approaches course.

Students use the school's library for research utilizing a link to the library University of West Bohemia, and to create a bibliography for their Internal Assessment. Also, the library has space and resources for teamwork on homework.

Other resources include:
P. Fannon, V. Kadelburg, B. Wooley, S. Ward: Mathematics, analysis and approaches, SL. Hodder Education
IB course books: Mathematics – analysis and approaches
Software: Graph, Desmos, Geogebra
GDC:
Texas Instruments TI-Nspire CX
Texas Instruments TI-84 Plus C
Casio fx-CG50
Web pages:
https://ibo.org/
https://www.khanacademy.org/math
https://www.symbolab.com/
https://www.geogebra.org/.geogebra.
www.padowan.dk